

Probability Distributions and Cumulative Functions

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1 Probability Distribution vs. Density Function

1.1 Discrete Random Variable: Die Roll (PMF)

Scenario: Fair 6-sided die roll

Random Variable: $X = \text{Outcome } (1,2,3,4,5,6)$

Probability Mass Function (PMF):

$$P(X = x) = \begin{cases} \frac{1}{6} & x = 1, 2, 3, 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

x	$P(X = x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Properties:

- $\sum P(X = x) = 1$
- $0 \leq P(X = x) \leq 1$

1.2 Continuous Random Variable: Human Height (PDF)

Scenario: Adult male heights

Random Variable: $H \sim \mathcal{N}(175, 10^2)$

Probability Density Function (PDF):

$$f(h) = \frac{1}{10\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{h-175}{10}\right)^2}$$

normal_pdf.png

Properties:

- $P(a < H \leq b) = \int_a^b f(h)dh$
- $\int_{-\infty}^{\infty} f(h)dh = 1$
- $f(h) \geq 0$

2 Cumulative Distribution Function (CDF)

2.1 Definition

The CDF of a random variable X is:

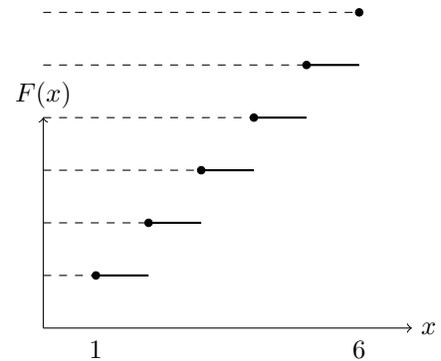
$$F(x) = P(X \leq x)$$

Works for both discrete and continuous variables.

2.2 Die Roll Example (Discrete CDF)

CDF Calculation:

$$\begin{aligned} F(1) &= P(X \leq 1) = \frac{1}{6} \\ F(2) &= P(X \leq 2) = \frac{2}{6} \\ &\vdots \\ F(6) &= P(X \leq 6) = 1 \end{aligned}$$



2.3 Bus Arrival Times (Continuous CDF)

Scenario: Bus arrival with average wait time = 10 minutes

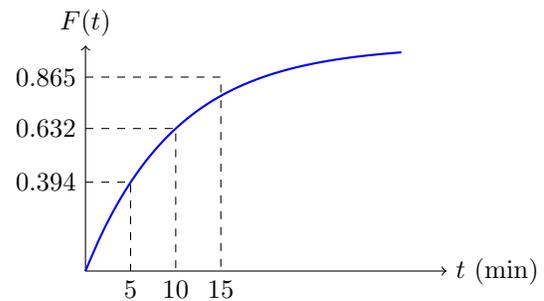
Random Variable: $T \sim \text{Exp}(\lambda)$

PDF: $f(t) = \lambda e^{-\lambda t}$ with $\lambda = \frac{1}{10}$

CDF: $F(t) = 1 - e^{-\lambda t} = 1 - e^{-t/10}$

Applications:

1. $P(T \leq 5) = 1 - e^{-0.5} \approx 0.394$
2. $P(T > 15) = e^{-1.5} \approx 0.223$
3. $P(5 < T \leq 15) = e^{-0.5} - e^{-1.5} \approx 0.383$



2.4 Why $\lambda = 1/10$?

- Exponential distribution mean = $\frac{1}{\lambda}$
- Given mean waiting time = 10 minutes:

$$E[T] = \frac{1}{\lambda} = 10 \implies \lambda = \frac{1}{10}$$

- Arrival rate: $\frac{1}{10}$ buses per minute

3 CDF Properties

- **Universal:** Applies to all random variables

- **Monotonic:** $x_1 < x_2 \implies F(x_1) \leq F(x_2)$
- **Limits:** $\lim_{x \rightarrow -\infty} F(x) = 0, \lim_{x \rightarrow \infty} F(x) = 1$
- **Interval Probability:** $P(a < X \leq b) = F(b) - F(a)$
- **Quantile Function:** $Q(p) = F^{-1}(p)$ (e.g., median $Q(0.5)$)